
NICE: Robust Scheduling through Reinforcement Learning-Guided Integer Programming

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Abstract

Integer programs provide a powerful abstraction for representing a wide range of real-world scheduling problems. Despite their ability to model general scheduling problems, solving large-scale integer programs (IP) remains a computational challenge in practice. The incorporation of more complex objectives such as robustness to disruptions further exacerbates the computational challenge. We present NICE (Neural network IP Coefficient Extraction), a novel technique that combines reinforcement learning and integer programming to tackle the problem of robust scheduling. More specifically, NICE uses reinforcement learning to approximately represent complex objectives in an integer programming formulation. We use NICE to determine assignments of pilots to a flight crew schedule so as to reduce the impact of disruptions. We compare NICE with (1) a baseline integer programming formulation that produces a feasible crew schedule, and (2) a robust integer programming formulation that explicitly tries to minimize the impact of disruptions. Our experiments show that, across a variety of scenarios, NICE produces schedules resulting in 33% to 48% fewer disruptions than the baseline formulation. Moreover, in more severely constrained scheduling scenarios in which the robust integer program fails to produce a schedule within 90 minutes, NICE is able to build robust schedules in less than 2 seconds on average.

1 Introduction

Scheduling is a ubiquitous type of discrete optimization problem that is often solved using integer programs (IPs) or mixed-integer programs (MIPs). While IPs and MIPs lend themselves to modeling a very wide range of scheduling problems, solving large-scale instances in practice can be computational challenging.

In most pragmatic applications, it is also important that the schedules be robust to uncertainties. Robust scheduling involves the building of schedules that will undergo minimal change when faced with unknown future disruptions. While MIP formulations of scheduling problems can be extended to represent robustness objectives, the resulting problems are often much more computationally challenging than their baseline, non-robust counterparts. Even with state-of-the-art solvers, such extensions to accommodate robustness can add hours to the time needed to compute an optimal schedule, sometimes making them impractical for real-world use.

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In this paper, we propose a technique, Neural network IP Coefficient Extraction (NICE), that seeks to find a quick-but-approximate solution to a scheduling problem with an additional robustness objective, by using reinforcement learning (RL) to guide the IP formulation. We first determine a *feasible* schedule using a baseline IP. We simultaneously train an RL model to build a schedule for the same problem, using a reward function that leads to more robust schedules (but that would have added considerable computational burden if encoded directly in the IP formulation). Then, rather than use the RL model to create a schedule directly, we use the probabilities in its output layer to assign coefficient weights to the decision variables in our simpler IP to create a feasible schedule. By doing so, we leverage the intuition behind knowledge distillation [1] that the distribution of values in the output layer of a neural network contains valuable information about the problem.

NICE allows us to approximate the robust scheduling objective function with significantly fewer variables and constraints. We find that, across a variety of disruption scenarios, our approach creates schedules with 33% to 48% fewer changes than than the baseline. Moreover, in certain practical problem instances, it is able to do so in a matter of seconds; the corresponding IP that explicitly optimizes for robustness fails to produce a solution within 90 minutes for the same scenarios.

The main contribution of this paper is the introduction of a new technique to approximate complicated IP formulations using RL. To the best of our knowledge, NICE is the first method to use information extracted from neural networks in IP construction. We illustrate the performance of NICE in the context of robust flight crew scheduling, by using NICE to create disruption-resistant crew schedules (i.e., assignments of pilots to flights).

2 Background and Related Work

Personnel scheduling has been a long-standing challenge in Operations Research, and has been the focus of much research over the past several decades [2, 3]. Integer programs (IPs) and mixed integer programs (MIPs)¹ have been widely-used for personnel scheduling, in large part due to their ability to represent general scheduling problems. However, despite the power of IPs to model scheduling problems, solving large-scale IPs in practice is often computationally challenging [4].

Most real-world applications also need robust schedules, namely, schedules that do not require considerable adjustments to personnel assignments in the event of an unforeseen disruption. While robust MIP-based formulations of scheduling problems can be developed, they are usually at least as computationally challenging as their non-robust counterparts [5, 6, 7, 8].

The scheduling of flight crews (e.g., pilots) is a personnel scheduling problem that arises in the context of aviation [9, 10, 11, 12]. Similar to other scheduling problems, crew scheduling has traditionally been tackled using large-scale IPs, both for airline and military flight crews [13, 14, 15]. Flight delays are the main cause of disruption to crew schedules in commercial aviation; buffers (or slack in the schedule) have therefore been considered as a mechanism to achieve schedule stability amidst flight delays [16, 17]. A *buffer* refers to the amount of time between the successive flights flown by a particular pilot. By increasing these buffers, a new pilot assignment is less likely to be needed due the initially-planned pilot being delayed, and therefore being unable to make the flight. In recent work, Chin [18] considers an IP-based formulation for the incorporation of such buffers into flight crew schedules, in the context of an application to an air force squadron.

2.0.1 Learning-Based Approaches to Scheduling

Successes in deep (reinforcement) learning have motivated research that focus on obtaining end-to-end solutions to combinatorial optimization problems; e.g., the travelling salesman problem [19, 20, 21, 22, 23] and the satisfiability problem (SAT) [24, 25]. RL has also been used in the resource management and scheduling domains on a diverse set of real-world applications. Gomes [26] uses an asynchronous variation of the actor-critic method (A3C) [27] to minimize the waiting times of patients at healthcare clinics. The lack of readily available optimization methods for this problem due to the ad-hoc nature of patient appointment scheduling motivates the usage of RL for this application to model the uncertainty. Mao et al. [28] introduce DeepRM which uses resource occupancy status in the form of images as the states and use neural networks for training the RL

¹We use the terms integer program (IP) and integer linear program (ILP) interchangeably unless noted otherwise; similarly for mixed-integer program (MIP) and mixed-integer linear program (MILP).

agent. Chen et al. [29] improve upon DeepRM by modifying the state-space, reward structure and the network used in the DeepRM paper. Chinchali et al. [30] use RL for cellular network traffic scheduling. They incorporate a history of the states observed to re-cast the problem as a Markov Decision Process (MDP) [31] from a non-Markovian setting. Along with this modification, they construct a reward function which can be changed by the operators according to preference. In all of the works above, the models used application-specific state space, action space and reward structures to optimize special-purpose objective functions to create schedules.

In recent work, Nair et al. [32] use a bipartite graph representation of a MIP and leverage graph neural networks [33, 34] to train a generative model over assignments of the MIP’s integer variables. In contrast to this approach, we do not use generative models to solve an IP formulation, but instead use RL to formulate the IP itself. Very recent work by Ichnowski et al. [35] uses RL to speed up the convergence rates in quadratic optimization problems by tuning the inner parameters of the solver.

2.0.2 Knowledge Distillation

Hinton et al. [1] explored the use of internal neural network values to distill the knowledge learned by a model. They reasoned that the probabilities in a neural network’s output layer carry useful information, even if only the maximum probability value is used for ultimate classification: “An image of a BMW, for example, may only have a very small chance of being mistaken for a garbage truck, but that mistake is still many times more probable than mistaking it for a carrot.”

In their work, Hinton et al. [1] used the values from the input to the output layer of a larger neural network (known as the logits), as well as the training data itself, to train a smaller neural network. This smaller neural network achieved fewer classification errors than a network of the same size trained only on the training data. They also used a similar approach to train a neural network on speech recognition data, this time training a network with the same architecture as a neural network trained on the data directly. They found that the new, distilled model performed better than the original one; it also matched 80% of the accuracy gains attained by averaging an ensemble of 10 neural networks with the same architecture, each initialized with different random weights at the beginning of training. Our approach uses the probabilities output by a neural network to extract objective function coefficient weights, leveraging the intuition of Hinton et al. [1].

3 Approach

3.1 Crew Scheduling Problem

In this paper, we seek to build robust schedules for a version of the flight crew scheduling problem (hereafter referred to as the “crew scheduling problem”, which has a long history in both commercial and military aviation [36, 37, 38]. We consider the scenario in which we are given a collection of flights that must be flown by a given squadron (i.e., a group) of pilots. Every flight has multiple slots, each of which must be filled by a different pilot. Each slot has qualification requirements that must be satisfied by any pilot who is assigned to that slot. Depending on their qualification, a pilot would only be eligible to fill a subset of slots. Finally, every pilot has some specified availability (or alternatively, a list of dates on which they are on “leave” and therefore cannot be scheduled to fly). We assume that a day is the time-discretization of our schedules, although this could be generalized to another choice of time unit. In other words, a feasible schedule assigns a specific pilot to every slot that is on every flight flown on each day in the scheduling horizon, such that the qualification requirements for the slots and the availability restrictions of the pilots are satisfied.

3.2 Baseline Integer Program Formulation

Chin [18] and Koch [15] created a baseline IP for the crew scheduling problem, producing a satisfying assignment with respect to all relevant constraints. We use a similar construction for our baseline IP, with the primary difference of having the decision variable for the assignment of each pilot to each slot rather than each pilot to each flight. We define the following sets and subsets:

$i \in I$	The set of pilots
$f \in F$	The set of flights
$s \in S$	The set of all slots
$U_f \subset F$	Flights that conflict with flight f
$L_i \subset F$	Flights that conflict with pilot i 's leave
$S_f \subset S$	The set of slots belonging to flight f
$Q_i \subset S$	Slots that pilot i qualifies for

We use the binary decision variable X_{is} , which is 1 if pilot $i \in I$ is assigned to slot $s \in S$, and 0 otherwise. We now have the following equation:

$$\max \sum_{i \in I} \sum_{s \in S} X_{is} \quad (1.1)$$

such that:

$$X_{is} = 0 \quad \forall i \in I, f \in L_i, s \in S_f \quad (1.2)$$

$$X_{is} = 0 \quad \forall i \in I, s \in S \setminus Q_i \quad (1.3)$$

$$\sum_{s \in S_f} X_{is} \leq 1 \quad \forall i \in I, f \in F \quad (1.4)$$

$$\sum_{i \in I} X_{is} = 1 \quad \forall s \in S \quad (1.5)$$

$$\sum_{s \in S_f} X_{is} + \sum_{s' \in S_{f'}} X_{is'} \leq 1 \quad \forall i \in I, f \in F, f' \in U_f \quad (1.6)$$

$$X_{is} \in \{0, 1\} \quad \forall i \in I, s \in S \quad (1.7)$$

Constraints 1.2 and 1.3 ensure that pilots are only assigned to slots that they are qualified for, and that pilots will never be assigned to flights that conflict with their leave. Constraint 1.4 prevents the scheduling of pilots to multiple slots on the same flight. Constraint 1.5 ensures that every slot gets filled by exactly one pilot. Constraint 1.6 prevents assignment of pilots to conflicting flights. We make our pilot-slot decision variable binary with Constraint 1.7. Finally, Equation 1.1 ensures that we fill as many slots as possible. Due to our requirement that each slot have exactly one pilot associated with it, this equation always produces the same objective value.

3.3 Buffer Formulation

Chin [18] utilizes additional constraints and decision variables to optimize schedules for robustness by increasing the amount of buffer time between flights. For our purposes, we consider the buffer time between two flights to be the number of full days between their start and end time. For example, say pilot X is assigned to flights A and B ; flight A ends on day 1, and flight B starts on day 5. We say that there is a buffer time of 3 days for pilot X in this instance, since there are 3 days between days 1 and 5, exclusive.

To incorporate buffers into an IP, Chin identifies all flight pairings that would create a buffer less than or equal to some maximum threshold, T_{buffer} , used to dampen the complexity of realistic problem instances. Then, the author creates a $\{0, 1\}$ decision variable $B_{iff'}$ for all pilots $i \in I$ and flights $f, f' \in F \times F$. The author uses constraints to ensure that $B_{iff'}$ is 1 if and only if the following conditions are met:

1. $f \neq f'$
2. Pilot i qualifies for at least one slot in both f and f' .
3. f and f' have a buffer between 0 and T_{buffer} , inclusive.

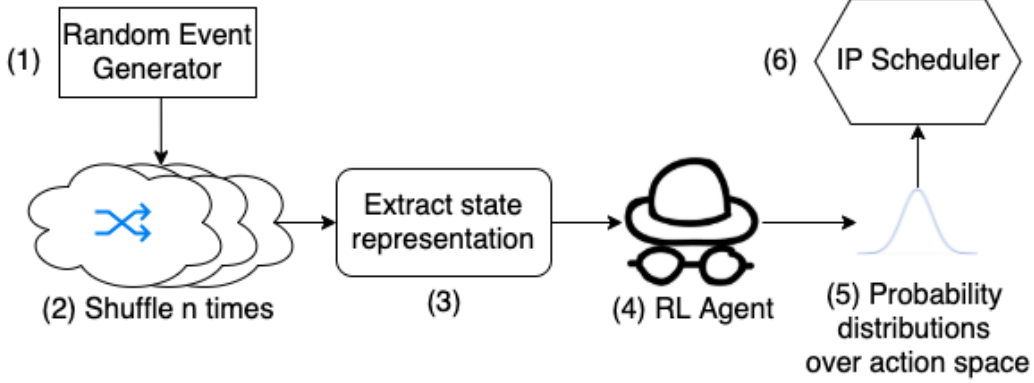


Figure 1: (1) Random events are generated using the process described in the Experiments Section. (2) Slots in the events are randomly shuffled if using the Monte Carlo Approach. (3) State representations are extracted for each event from the given set of randomly generated events. (4) The states are given as input to the RL agent. (5) Create n different schedules using the RL agent. (Note that these schedules are not the final schedules.) (6) The probability outputs at each timestep are used in the objective function for the IP formulation as given in Equation 3 to create the final schedules using the IP solver

4. Pilot i is assigned to both f and f' , without any flights in between.

The author then defines a buffer penalty $b_{iff'} \in [-1, 0)$ that gets increasingly negative for higher buffers: down to -1 when the buffer between f and f' is 0, and closest to 0 when the buffer is T_{buffer} . Note that buffers longer than T_{buffer} effectively have a penalty of 0. Finally, they incorporate all of this into the following objective function to optimize buffer time across the schedule:

$$\max \sum_{i \in I} \sum_{f, f' \in F \times F} b_{iff'} B_{iff'} \quad (2)$$

In our buffer IP, we incorporate this formulation with the assistance of additional auxiliary $\{0,1\}$ decision variables for each pilot-flight combination, setting it to 1 if a pilot is assigned to any slot on a given flight, and 0 otherwise. In our experiments, we found that, in certain problem instances, the buffer IP was highly effective in producing robust schedules.

Chin’s work also introduced an integer program formulation that incorporated the concept of move-up crews [39] to increase robustness. However, in our testing, this integer program did not produce particularly robust schedules, so we focused our attention on buffers.

3.4 Reinforcement Learning Formulation

We model the building of a valid schedule with a discrete event simulation (DES): at each time step, we take an action, which affects the state of our system. The main idea of our reinforcement learning scheduling approach is to order the slots that need to be scheduled and, at each slot, pick a pilot to assign to that slot. This technique, previously used by Washneck et al. [40] for production scheduling, gives us an action space size equal to the number of pilots. If we get through all of the slots, we end up with a complete schedule, though filling all of the slots is not a guarantee; the RL scheduling agent could back itself into a corner, leaving no pilots to assign to a given slot based on its previous decisions. We choose to use Proximal Policy Optimization (PPO) [41] with a discrete action space to train deep neural networks for actor and critic network estimation. The actor network gives a probability distribution over the pilots to choose given the state input.

3.5 NICE

In our early exploration, when building schedules with non-robustness objectives, the RL-produced schedules would often approach the effectiveness of the IP schedules with respect to the optimized metrics, but they would never do better. This observation gave us two key insights. First, the RL

model assigns pilots to slots sequentially, operating almost in a greedy fashion. Granted, it is trying to maximize its expected total discounted reward, but once it makes an assignment, it cannot change it. Further, the RL agent does not have a perfect view of the full schedule in its state space, meaning it has imperfect information at the time of each pilot assignment. This sequential decision making lacks in comparison to the global view of the IP scheduling approach, where we optimize an objective function across all pilot assignments, factoring in the complex interplay of related constraints. Second, it was clear that our RL scheduling agent was capable of learning. While it could not match the performance of the IP schedules in our preliminary exploration, it still produced schedules with considerably better objective performance than the baseline. When scheduling objectives can easily be captured in an IP, this observation is not particularly helpful. However, for more complicated optimizations that integer programming struggles with, this insight proves useful: to avoid the greedy pitfalls of RL for scheduling while still leveraging the knowledge learned by our neural network, we can use the probabilities produced by the output layer in our IP scheduling formulation.

These two observations motivated the creation of NICE. NICE uses RL to approximate sophisticated integer programs with a simpler formulation. We can use this technique to build an approximately-optimal solution to the crew scheduling problem. We set up our baseline integer program to build a schedule, but modify the objective function given in Equation 1.1. The objective function maximizes the number of pilots assigned across the schedule but, because of our constraint that assigns exactly one pilot to each slot (Constraint 1.5), our objective value is fixed across all satisfying pilot-slot assignments. Our decision variable, X_{is} , is 1 if pilot i is assigned to slot s and 0 otherwise. This variable aligns neatly with our RL scheduling formulation, which considers slots in a fixed order. At each slot, it produces a probability for each pilot that captures how likely assigning that pilot to that slot is to maximize the expected total discounted reward in a given scheduling episode. It then assigns the pilot with the maximum probability to that slot. We can use a_{is} to refer to the probabilities output by the network for the assignment of pilot i to slot s . Now, to leverage the knowledge learned by the RL scheduling approach in our IP formulation, we can incorporate a_{is} into our objective function:

$$\max \sum_{i \in I} \sum_{s \in S} a_{is} X_{is} \tag{3}$$

With this new equation, our IP scheduling approach is incentivized to pick the pilot with the highest probability possible at each slot, only selecting pilots with lower probabilities if constraints forbid such an assignment. This new formulation approximately captures the reward function used by the RL agent while giving it a global view of pilot assignment.

3.6 Extracting Probability Weights

An important issue we had to address was the extraction of a_{is} from our RL neural network. The actions taken at each state of the RL scheduling process can impact the pilot probability vector at later states. Thus, the order that the RL scheduler fills the slots has a potentially problematic impact on the a_{is} weights. Simply running the RL scheduling process as normal on a given problem instance while extracting the probability vector at each slot could cause the specific actions taken to bias our a_{is} values, diminishing the advantage of the IP scheduler’s global outlook.

In our experiments, we used two different approaches. In the first one, we took a Monte Carlo approach, trying to approximate the average weight across all possible orders of scheduling the slots. In this approach, we randomly shuffled the order of slots that the RL scheduler had to assign and then recorded the a_{is} weights at each step of the scheduling process. We ran this process n times to get n total a_{is} values for each pilot-slot pair. If the RL agent could not fill a slot in that round of scheduling, we set the a_{is} value to 0. We then averaged the n values for each pilot-slot pair to get our final a_{is} values. Note that this causes the scheduling process to take longer for higher values of n because it has to run more RL scheduling rounds.

The next approach, which we call the “blank slate” approach, exploited the fact that the probability weights for the pilots produced by the first slot are not dependent on any previous actions taken. Thus, we can make each slot our first slot to get weights that are not dependent on previous decisions made. To do so, for each slot s in our fixed order, we initialized a new RL agent with the same underlying neural network, cutting out all of the states that occur before s then extracting the a_{is} values for the pilots on that slot.

3.7 Incorporating Robustness

To use NICE to build robust schedules, we train an RL scheduler to optimize buffer time in its assignments. To do so, we include a reward of $b + 1$ whenever the RL agent places a pilot on an event that forms a buffer of length b with the pilot’s most recent event. We add $+1$ term to reward the agent for making a placement, regardless of buffer. We give the agent a reward of $T - 1$ when it places a pilot with no previous events scheduled. We chose $T - 1$ because this is the maximum reward for the T -day schedules that the RL agent trains on. For example, imagine pilot X is assigned to a 0-day flight starting and ending on day 1. If the pilot were assigned to a flight starting on day 7, that assignment would earn a reward of 6, because there are 5 days between day 1 and day 7, exclusive. We do not use a maximum buffer value like T_{buffer} in the IP formulation because larger T_{buffer} values do not noticeably affect the run time of our program like it does with the IP. We included two exceptions to this reward policy. First, to incentivize building full schedules, the agent earns a reward of 25 when all slots in an episode are scheduled. Second, to deter the creation of schedules, we give the agent a reward of -10 when it is unable to schedule all events in an episode.

4 Experiments

To train our RL agent, we created an OpenAI Gym [42] type environment for the crew scheduling problem. We utilized an anonymized dataset from a US Air Force flying squadron to construct a random event generator. The dataset contained 87 pilots with 32 different qualifications. It also consisted of a log of flights flown over about 6 months which amounted to a total of 801 flights, each containing between 2 and 3 slots. There were 16 different total types of flights, where the type determined the qualification requirements for the slots on the flight. These flights were subdivided into two categories: missions and simulators, which we treat equivalently except for the purposes of our random event generation. There were 7 mission types and 9 simulator types. We trained our RL agent on randomly generated flights based off of this dataset. To generate the random flights, we first divided our dataset into simulators and missions that started in 26 different full-week intervals. For each week, to generate a random set of events, we created α random mission based flights, where α is drawn from a normal distribution with a mean and standard deviation equal to the mean and standard deviation of missions across all 26 weeks. Similarly, we also create β random simulator-based flights, where β is drawn from a similar distribution that uses simulators instead of missions. The dataset also contained a variety of training requirements that each pilot, ideally, would fulfill; the number of times the pilot should fulfill each requirement; and information about which flight satisfied which training requirements. Along with the training requirement information, each flight contained two binary training requirement qualifiers (TRQs) to help determine which training requirements the flight fulfilled. Note that we did not use the training requirement information outside of the state space formulation for our RL scheduler.

For each flight generated, we picked a random day in the scheduling week for it to start. We then randomly pick a type for the flight, where the probability of picking that type of mission was proportional to the number of times it showed up in the dataset. To determine the length of duration for the flight, we randomly sample a flight of the selected type from the dataset and use the length of duration of sampled flight’s length. We followed an identical process to generate each simulator, except each simulator started and ended on the same day, so we did not randomly pick a length. The dataset provides the dates each pilot is on leave, which we used directly. Then we created a fixed ordering of slots that need to be scheduled. We order the slots first by the corresponding flight’s start date and use the flight’s arbitrary unique ID as a tie-breaker. We order slots of the same flight by ascending qualification. The assignment of a pilot to a slot serves as a time-step in our DES.

The state space for the RL agent includes:

1. The pilots available for the current slot, encoded as a binary vector.
2. A flattened vector encoding the current event to be scheduled, consisting of:
 - (a) A one-hot encoding of the event type
 - (b) A binary vector indicating whether each TRQ was true or false.
 - (c) A binary vector representing the pilots assigned to the current event. (Note that each event may have 2-3 pilot slots)
 - (d) The event duration (in days)

% Flights Delayed	Number of Disruptions				Significance of Difference (p -value)		
	NICE	Baseline IP	RL	Buffer IP	NICE-Baseline	NICE-RL	NICE-Buffer
25	0.34 ± 0.71	0.61 ± 1.07	32.6 ± 7.33	0.0 ± 0.0	0.03	< 0.01	< 0.01
50	0.67 ± 0.92	1.16 ± 1.55	27.1 ± 7.13	0.0 ± 0.0	< 0.01	< 0.01	< 0.01
75	0.66 ± 0.99	1.13 ± 1.73	23.0 ± 6.34	0.0 ± 0.0	0.01	< 0.01	< 0.01
100	0.63 ± 0.82	1.06 ± 1.50	17.9 ± 6.29	0.0 ± 0.0	0.01	< 0.01	< 0.01

Table 1: Average and standard deviation of disruptions across scheduling methods when flights are delayed (lower values are better). Scheduling density of 1.

- (e) The number of days between the start of the scheduling episode and the start of the event
 - (f) The number of days between the start of the scheduling episode and the end of the event
3. A vector containing the total number of training requirement fulfillments each pilot could receive for flying that event, if it were flown a sufficient number of times

We built our PPO code on top of OpenAI’s SpinningUp library [43] in PyTorch [44]. To build the neural network for our NICE scheduler, we trained a variety of models with different hyperparameter combinations. One of the hyperparameters of particular interest was the training schedule density, d . Recall that we scheduled α flights and β simulators in a round of scheduling. During training, we multiplied α and β by d so the agent would schedule more flights in the same time period. We trained RL models with $d \in \{1, 2, 3\}$. We trained a model with 5 different seeds for each value of d , ultimately creating $3 \times 5 = 15$ different neural networks for testing. We used the default values for all PPO related hyperparameters used in the SpinningUp repository. Each model was trained for 5,000 epochs on a computer with 40 Intel Xeon-P8 CPUs. We utilized the MPI [45] version of the PPO implementation in SpinningUp to run 40 parallel processes to train the model. Source code for our experiments is available at <https://github.com/nsidn98/NICE>

4.1 Scheduling Parameter Selection

For further experimentation, we had to pick the best combination of RL model and weight extraction method to use. We parameterize our weight extraction methods with the variable n , where $n = 0$ represents the “blank slate” extraction method mentioned previously, and $n > 0$ represents the n value used in the Monte Carlo approach.

To select the best combination, we built an environment where, using the same flight generation process to train the RL scheduler, we generate 1 week’s worth of flights. From these flights and associated slots, we generate pilot-slot pairings using the baseline integer program and the NICE schedule with weights extracted from the selected RL network and the given n value. Next, one day into the schedule, we delayed 50% of the flights that had not already left. We pushed back each flight by a number of days randomly chosen uniformly between 1 and 3, figuring that delays longer than 3 days were relatively rare. To fix this disruption in both of these schedules, we used an integer program that minimized the number of changes to the pilot-slot pairings. From this disruption resolver, we end up with the number of disruptions that occurred under each schedule. Sometimes, we would randomly generate a series of flights that made it impossible for the IP or NICE approach to schedule because a pilot-slot pairing did not exist that met all of the constraints. In these cases, we skipped to the next series of randomly-generated flights, not recording any disruption data because there was no schedule to disrupt. Then, for each neural network and for each n value $n \in \{0, 2, 4, 8\}$ (60 experiments total), we ran this scenario 20 times, comparing the average number of schedule disruptions between the baseline IP and the NICE scheduler across those 20 runs. We ran these experiments, and all subsequent ones, on an Intel Core i7 laptop with 32 GB of RAM, solving the IPs with Gurobi Optimizer [46], version 9.1.2.

We calculated the ratio of disruptions, r , between the two methods, where $r < 1$ indicates that the NICE method provided a schedule with fewer disruptions on average than the IP baseline. We then determined the median r value over seed values for each training density and weight extraction

% Flights Delayed	Number of Disruptions			Significance of Difference (p -value)	
	NICE	Baseline IP	RL	NICE-Baseline	NICE-RL
25	1.99 ± 1.99	2.99 ± 2.68	59.2 ± 13.1	< 0.01	< 0.01
50	3.17 ± 2.27	5.01 ± 4.33	51.2 ± 12.2	< 0.01	< 0.01
75	3.32 ± 2.33	6.32 ± 5.64	43.1 ± 12.4	< 0.01	< 0.01
100	2.81 ± 2.23	4.70 ± 4.49	35.4 ± 10.5	< 0.01	< 0.01

Table 2: Average and standard deviation of disruptions across scheduling methods when flights are delayed (lower values are better). Scheduling density of 2.

method combination. We used the lowest median r value to determine our final scheduling method and underlying neural network for NICE. As a result of this process, we obtained a network trained with a density of 1 and an n value of 2. In this case, across seed values, the three networks produced the same median value for this combination, so we arbitrarily chose a network from those three models. The median r value was 0.25, with a range of 0.08 (0.25 to 0.33). We used this network and n value in our further experiments.

Overall, the combinations had fairly stable performance over seed values, with the highest range in r across seeds being 0.81. Notably, the n value of 0 produced the 3 highest ranges (0.81, 0.69, and 0.56), indicating that the “blank slate” weight extraction method led to a wider range of disruptions in produced schedules across random seeds.

4.2 Baseline Scheduling Performance

To show the efficacy of NICE scheduling, we ran our best scheduling combination on the same disruption scenario, this time using different percentages of flights delayed, f . We selected f values of 25%, 50%, 75%, and 100%. We ran each scenario 100 times. We also built a schedule using our RL agent directly and with the buffer integer program (with $T_{\text{buffer}} = 4$ days) for comparison. We show the results of our buffer-rewarded NICE scheduler in Table 1. During these runs, the buffer IP and NICE scheduler had similar run times, both averaging less than 0.85 seconds to create each schedule.

We note that, because the constraints are exactly the same between the IP and NICE scheduling approaches, we skipped recording the disruption value for the IP scheduler if and only if we also skipped the disruption value for the NICE scheduler. Because of this alignment, the p -values comparing the two methods were obtained using a 2-tailed dependent t-test for paired samples between the NICE and IP schedulers. This was not the case for the RL approach, which can generate a partial schedule, stopping when it is not able to assign a pilot to the next slot due to its previous decisions. When the RL approach created a partial schedule, we did not record its performance on that schedule to include in the average because it was unable to produce a full schedule like the baseline IP and NICE schedulers. For this reason, in the NICE vs RL comparison, we use Welch’s t-test for independent samples.

4.3 Highly-Constrained Scheduling Scenarios

In the schedule disruption scenario that we considered, the buffer IP formulation had little trouble building a robust schedule in a reasonable amount of time. However, the time advantages of NICE become apparent in a more constrained scheduling setting. To demonstrate this efficiency, we performed the same experiment on NICE, averaging over 100 trials. This time, though, we used a scheduling density of 2, creating twice the number of flights on average in each round of scheduling. This scenario is realistic in settings where, due to outside factors, many flights must be filled. For timing reasons, we only compared NICE with the baseline IP and the pure RL scheduler. We show the results for this experiment in Table 2. Importantly, across all flight delay percentages, the NICE scheduler took an average of between 1.85 and 1.90 seconds to build a schedule, with a standard deviation between 0.55 and 0.60 seconds. We ran the exact same experiment for each disruption percentage for 10 iterations with the buffer-optimizing IP. However, we ended each experiment after 90 minutes, at which point the buffer IP had not finished building a single schedule.

5 Discussion

Our results clearly show the advantages of the NICE approach. In the baseline scheduling scenario, NICE produced schedules that provided 40% to 45% of the disruption reduction of the buffer IP compared to the baseline IP. In a powerful display of its usefulness, in a dense scheduling environment, NICE performed 33% to 48% better than the baseline IP, producing 100 schedules with an average time of less than 2 seconds while the buffer IP failed to produce a single schedule in 90 minutes. In all of these experiments, the NICE scheduler massively outperformed the RL scheduler from which it was derived. These outcomes indicate that NICE can harness various advantages of IP and RL scheduling to build a hybrid approach that improves on both methods used independently.

In the less-constrained baseline scheduling environment, NICE did not compete favorably against the buffer IP, though it still did better than the baseline. In this scenario, in a similar amount of time as NICE, the buffer IP produced perfect schedules with no disruptions after flights were delayed. This result highlights the ideal use-case for NICE: situations where approximate results are useful and the size of the integer program makes it infeasible to solve in a reasonable amount of time. The more constrained (density = 2) scenario fits this description well; the high number of flights in a shorter interval created more constraints and variables than our IP solver could handle. By contrast, NICE was able to produce a robust schedule in under 2 seconds, on average.

5.1 Conclusions and Future Work

We introduced NICE, a novel method for incorporating knowledge gleaned from reinforcement learning into an integer programming formulation. We applied this technique to a robust optimization problem, looking at the assignment of pilots to flights so as to minimize schedule disruptions due to flight delays. We used NICE to build a scheduler for this problem where the RL agent proposes weights for the selection of crew members and the IP assigns the crew members using those weights. We performed experiments on the crew scheduling problem. Our results show that NICE outperformed both the baseline IP and the RL scheduler in creating schedules that are resistant to disruptions. Furthermore, in certain practical environments that caused the robust scheduling (buffer IP) formulation to be prohibitively slow, NICE was able to create a robust schedule in a matter of seconds.

The introduction of nonlinear objectives or constraints significantly deteriorates the computational performance of MIP and IP solvers². However, the reward structure used to train an RL agent is not bound by such restrictions. While we used NICE to approximate several linear constraints and additional variables in an IP, it would be interesting to see how the approach performs when approximating a nonlinear objective that cannot be formulated as an integer linear program. Next, to the best of our knowledge, IP or MIP formulations have previously not used information derived from a neural network to determine their objective functions. We applied this technique to the domain of robust scheduling, and found that it could efficiently produce robust schedules. Given that IPs have long been a mainstay of discrete optimization, we believe that these results indicate that NICE could be useful in addressing other scheduling problems.

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²The underlying formulations are no longer integer linear programs or mixed-integer linear programs

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A Technical Appendix

A.1 Move-up Crews

During our experimentation, we considered another factor that can lead to more robust schedules, move-up crews [39]. We found that move-up crews, even when optimally scheduled, did not create particularly robust schedules, but we include our experimental results here for reference. For the sake of clarity, because we dealt with individual pilots rather than crews, we will depart from the literature and use the term move-up *pilot* rather than move-up crew. A move-up pilot is someone who, by nature of their qualification and one of their scheduled flights, is readily available to move up to another flight should someone on that flight become unavailable, perhaps due to a delayed flight.

A.1.1 IP Formulation

We created an IP to increase move-up pilots in our schedules. We first define a threshold, T_{move} , for how far out we should look for move-up pilots. Now, we give a more formal definition of a move-up pilot: pilot $j \in I$, assigned to flight $g \in F$, is a move-up pilot for slot $s \in S_f$ on flight $f \in F$ if and only if all of the following conditions hold:

1. $f \neq g$
2. Flight g starts at the same time as or later than flight f , and no later than T_{move} days after f starts.
3. Flight g ends at the same time as or later than flight f .
4. Pilot j is not on leave that overlaps with f .
5. Pilot j is not scheduled to any flights that start before f and overlap with f .
6. Pilot j is qualified for slot s .

Using constraints, we define the binary decision variable $M_{jg,fs}$ to be 1 if pilot j assigned to flight g is a move-up pilot for slot s on flight f . We then use additional variables to build an objective function to maximize the number of move-up pilots in our final schedule. We can achieve this with the following objective function:

$$\max \sum_{i \in I} \sum_{f, g \in F \times F} \sum_{s \in S_f} M_{jg,fs} \quad (4)$$

A.1.2 RL Training

To train an RL scheduler to optimize for move-up pilots, we followed the same experimental procedure for the buffer-optimized RL scheduler, but we changed the reward function. For move-up pilots, we give a reward of $m + 1$ whenever our agent assigns pilot j to a slot on flight g , where m is the number slots on other flights that pilot j can serve as a move-up pilot for. We use a maximum move-up time (like T_{move}) of 2 days. We did not do so out of concern for program run time; on a practical level, moving a pilot to a flight any more than 2 days earlier would cause a significant burden on the pilot rather than supply the convenient schedule alleviation that move-up pilots are supposed to provide. Just like the reward function for buffers, we include a -10 penalty for incomplete schedules and a +25 reward for complete schedules. We trained 15 total neural networks with the move-up pilot reward structure, using 5 different random seeds and schedule densities of 1, 2, and 3.

A.1.3 Model Selection

We followed the same procedure as the buffer-rewarded networks to select the best network and n value to use for our weight extraction method. We used n values of 0, 2, 4, and 8. We used this procedure to obtain r , the ratio of average disruptions in the baseline IP schedule to average disruptions in the NICE schedule with the chosen parameters. $r < 1$ indicates that the NICE schedule had fewer disruptions on average. The best median r value was 0.46, with a range of 0.38 (0.25 to 0.63), produced with a scheduling density of 2 and an n value of 2. Two networks with these parameters and different seed values produced the median value, so we picked one arbitrarily. We used this model in our subsequent experiments.

The range across seeds for our move-up-rewarded NICE scheduler (0.38) was notably higher than the range for our best buffer-rewarded scheduling method, which was 0.08. Like the buffer-rewarded schedulers, the n /density combinations also had fairly stable performance across random seeds. The 3 highest ranges across random seeds were 0.88, 0.69, and 0.68. Similar to the buffer-rewarded schedulers, the 2 highest ranges were produced by the “blank slate” weight extraction method ($n = 0$).

% Flights Delayed	Number of Disruptions				Significance of Difference (p -value)		
	NICE	Baseline IP	RL	Buffer IP	NICE- Baseline	NICE- RL	NICE- Buffer
25	0.63 ± 0.93	0.61 ± 1.07	32.6 ± 7.42	0.33 ± 0.63	.88	< .01	< .01
50	1.05 ± 1.13	1.16 ± 1.55	27.6 ± 7.24	0.68 ± 0.90	.51	< .01	< .01
75	1.19 ± 1.25	1.13 ± 1.73	23.9 ± 6.44	0.74 ± 1.04	.73	< .01	< .01
100	1.11 ± 1.14	1.06 ± 1.50	19.1 ± 6.47	0.65 ± 0.88	.76	< .01	< .01

Table 3: Average and standard deviation of disruptions across scheduling methods when flights are delayed (lower the better). Scheduling density of 1. The NICE and RL schedulers used the move-up reward function in their underlying neural network.

A.1.4 Baseline Scheduling Performance

Using the selected model and n value, we ran the same disruption scenario as the buffer-rewarded NICE scheduler over 100 iterations. We compared the NICE scheduler against the baseline IP scheduler, the RL scheduler with the same underlying neural network, and the move-up IP scheduler with a T_{move} value of 2. The results are shown in Table 3.

A.1.5 Discussion

Based on our results, the move-up IP scheduler produced more robust schedules than any of the other methods, but it did not perform as strongly as the buffer IP scheduler, which entirely eliminated schedule disruptions across all percentages of flights delayed in our previous experiment. This weakness likely explains the performance of the NICE scheduler based on the move-up reward function, which showed no significant difference in schedule disruptions compared to the baseline IP scheduler. Because of the relative inefficacy of increasing the number of move-up pilots in producing robust schedules, we decided to focus our efforts on using buffers to reduce disruptions in our main work.